**Physics I**
**ISI B.Math**
**Final Exam : December 7, 2011**

Total Marks: 100
Answer any 5 questions. All questions carry equal marks.

**1.**A particle of mass $m$ moves along a trajectory given by $x = x_0 \cos \omega_1 t$ and $y = y_0 \sin \omega_2 t$.
**(a)**Find the $x$ and $y$ components of the force.Under what conditions is the force a central force? **(5)**

**(b)**Find the potential energy as a function of $x$ and $y$. **(5)**

**(c)** Find the kinetic energy of the particle and show that the total energy is conserved. **(5)**

**(d)** The equation of motion of a point electric charge of charge $e$ and mass $m$ in the field of a magnetic monopole of strength $g$ at the origin is

$$m\ddot{\mathbf{r}} = -ge\frac{\mathbf{r} \times \dot{\mathbf{r}}}{r^3}$$

Show that the kinetic energy $T = \frac{1}{2}m(\dot{\mathbf{r}})^2$ is a constant of motion.**(5)**

**2.(a)** In a one-dimensional elastic collision between two particles,in the laboratory frame, the relative velocity before collision is negative of the relative velocity after the collision. **(7)**

**(b)** Show that the above result also holds true in the zero momentum or centre-of mass frame **(2)**

**(c)** A ball of mass $m$ and initial speed $v_0$ bounces back and forth between a fixed wall and a block of mass $M$ with $M \gg m$ as shown in Figure 1. Assume that the ball bounces elastically and instantaneously. The coefficient of kinetic friction between the block and the ground is $\mu$. There is no friction between the ball and the ground.Assume that the distance to the wall is large enough so that the block comes to rest by the time the next bounce occurs. Show that if $v_i$ is the speed of the ball after the $i$th bounce,

$$v_{i+1} \approx (1 - 2\epsilon)v_i$$

to first order in $\epsilon = \frac{m}{M}$**(7)**
Hence find the speed after the $n$th bounce in terms of $v_0$, to the same order of approximation.**(4)**
(Hint: Use the result in part (a))

**3.** A particle of mass $m$ slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical. The half angle at the tip is $\alpha$. (See Fig. 2). Let $r$ be the distance from the particle to the axis, and let $\theta$ be the angle around the cone.
**(a)** Show that the Lagrangian of the particle in terms of the generalized coordinates $(r, \theta)$ is given by

$$L = \frac{1}{2}m\left(\frac{\dot{r}^2}{\sin^2\alpha} + r^2\dot{\theta}^2\right) - \frac{mgr}{\tan\alpha}$$

Identify the cyclic coordinate. **(7)**

**(b)** Find the equations of motion. Show that the generalized momentum associated with the cyclic coordinate is conserved. What is the physical meaning of this conserved quantity ? Can you identify the symmetry associated with this conservation law? Show that $\dot{\theta}$ can never change sign.**(8)**

**(c)**If the particle moves in a circle of radius $r = r_0$, find the angular frequency $\omega$ of this motion **(5)**.

**4.(a)**Find the number of degrees of freedom for the following systems **(6)**
(i) a door swinging on its hinges
(ii) A bar of soap sliding on the inside of a hemispherical basin
(iii) A rigid rod sliding on a flat table
(iv) four rigid rods flexibly joined to form a quadrilateral that can slide on a flat table.
(v) a rigid body of arbitrary shape moving in 3D
(vi) a diatomic molecule modeled as two spheres joined by a rigid rod.

**(b)** A rigid body of arbitrary shape is suspended about a pivot at a point $O$ and is free to swing in a vertical plane about an axis passing through $O$ as shown in figure 3. The centre of mass of the body is at $G$.

(i) Write down the equation of motion for the coordinate $\theta$ and from it show that the system is completely equivalent to a simple pendulum of length $L = \frac{k_O^2}{h}$ where $k_O$ is the radius of gyration of the rigid body about $O$, and $h$ is the distance between $O$ and $G$. **(6)**

(ii) Show that time period of oscillation will remain unchanged if the body is suspended about the point $O'$ (at a distance $h' = L - h$ from G, refer to figure 3) instead of $O$.**(8)**

**5.** **(a)** Two stars $S_1$ and $S_2$ of mass $m$ and $2m$, forming a binary star pair move under their mutual gravitational interaction force (refer to figure 4).Show that the equation of motion for the relative coordinate vector $\mathbf{r}$ is given by **(7)**

$$\ddot{\mathbf{r}} = \frac{3G}{r^2}\hat{\mathbf{r}}$$

**(b)** If it is known that the orbit of $S_1$ relative to $S_2$ is an ellipse with $S_2$ at one focus,show that i)the orbits of $S_1$ and $S_2$ in the zero momentum frame are similar ellipses, each with its focus at $C$, where $C$ is the centre-of-mass ii) the ratio of their major axes is $2:1$ and iii) they have the same time period.**(8)**

**(c)**State how Kepler's first law is modified when the sun-earth motion is considered to be a 2-body problem as opposed to the 1- body problem of the earth moving under the influence of a central force with the sun fixed at the centre of the force. **(5)**

**6.(a)** Two blocks of masses $m$ and $2m$ are connected by a light inextensible string which passes over a uniform circular pulley of radius $a$ and mass $4m$. Find the upward acceleration of the mass $m$. **(8)**

**(b)** A mass $m$ travels perpendicular to a stick of mass $m$ and length $l$, which is initially at rest. At what location should the mass collide elastically with the stick, so that the mass and the centre of the stick move with equal speeds after the collision? Find the time it takes the stick to make half a revolution about its centre after the collision. Moment of inertia of the stick about its centre is $\frac{ml^2}{12}$. **(12)**

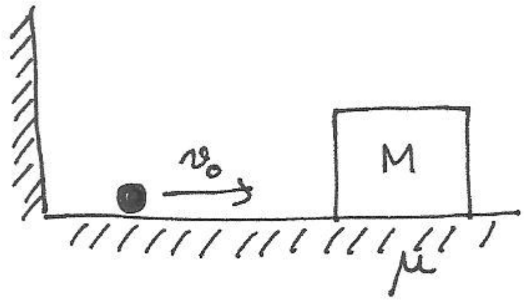


Fig. 1

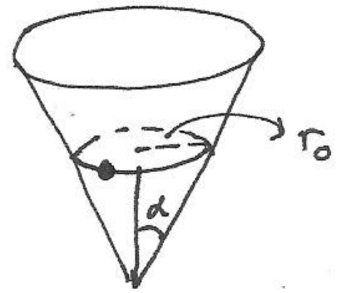


Fig. 2

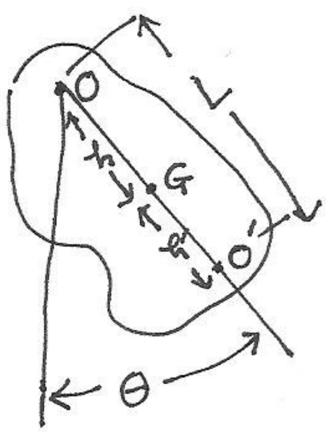


Fig. 3

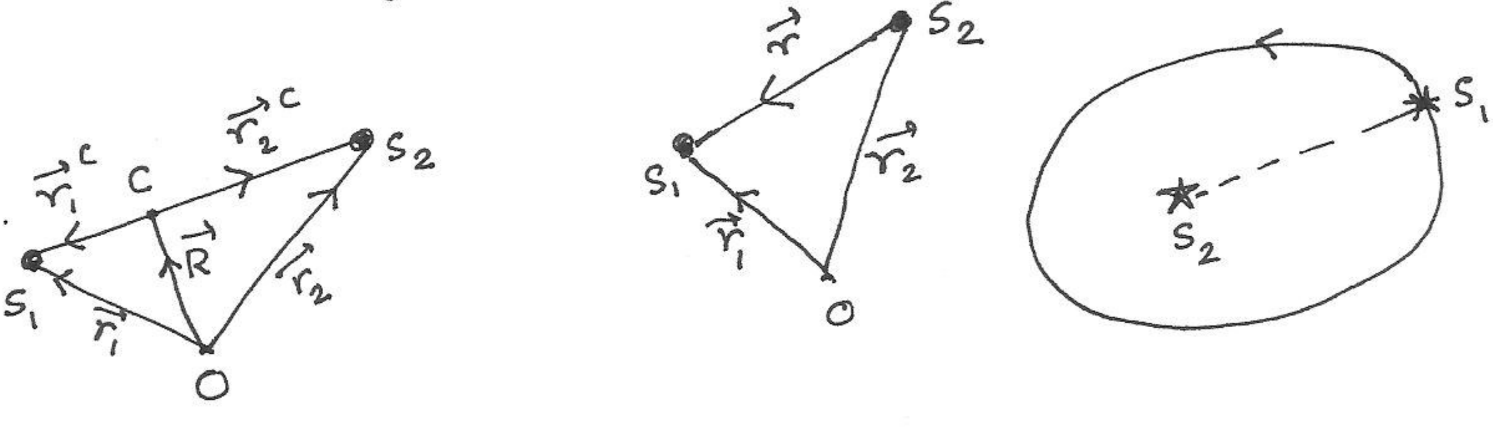


fig. 4